Abstract

A complementary set of sequences is that for which the sum of the aperiodic auto-correlation functions across all sequences in the set is zero except at the zero shift. They find utility in a wide range of applications, including that of Orthogonal Frequency Division Multiplexing (OFDM), a method of transmitting data simultaneously over a large number of frequencies. Sequences for OFDM have an associated parameter called the 'peak-to-mean envelope power ratio' (PMEPR), and sequences from small complementary sets have low PMEPRs, and so are attractive for engineering reasons.

This thesis follows up aspects of recent results of Davis and Jedwab, and Paterson, who give explicit constructions for complementary sets of sequences of length 2^m over $\mathbf{Z}_q$ (q even). Using techniques deriving from the algebraic normal form of the generalized Boolean functions that represent the sequences, a number of results concerning auto- and cross-correlations are proved, resulting in classes of sequences for which: the auto-correlation functions are the same; pairs of cross-correlation functions sum to zero, or are the same; pairs of cross-correlation functions sum to zero except at one shift; pairs of auto-correlation functions sum to zero except at the zero shift and one other shift. These results are then used to prove a conjecture of Paterson (concerning the maximum PMEPR of certain sets of sequences) for two specific cases, and to show that it is true in many others. A new lower limit on PMEPR is also developed that shows that the conjecture cannot be true in general.

Complementary sets are also manufactured from pairs of sequences given by the construction, thus demonstrating the structure that is inherent within such a pair. By examining the effect of the inverse Gray map on algebraic normal form, it is shown that a complementary pair from the construction over $\mathbf{Z}_2$ remains a complementary pair when mapped to $\mathbf{Z}_4$ by the inverse Gray map.